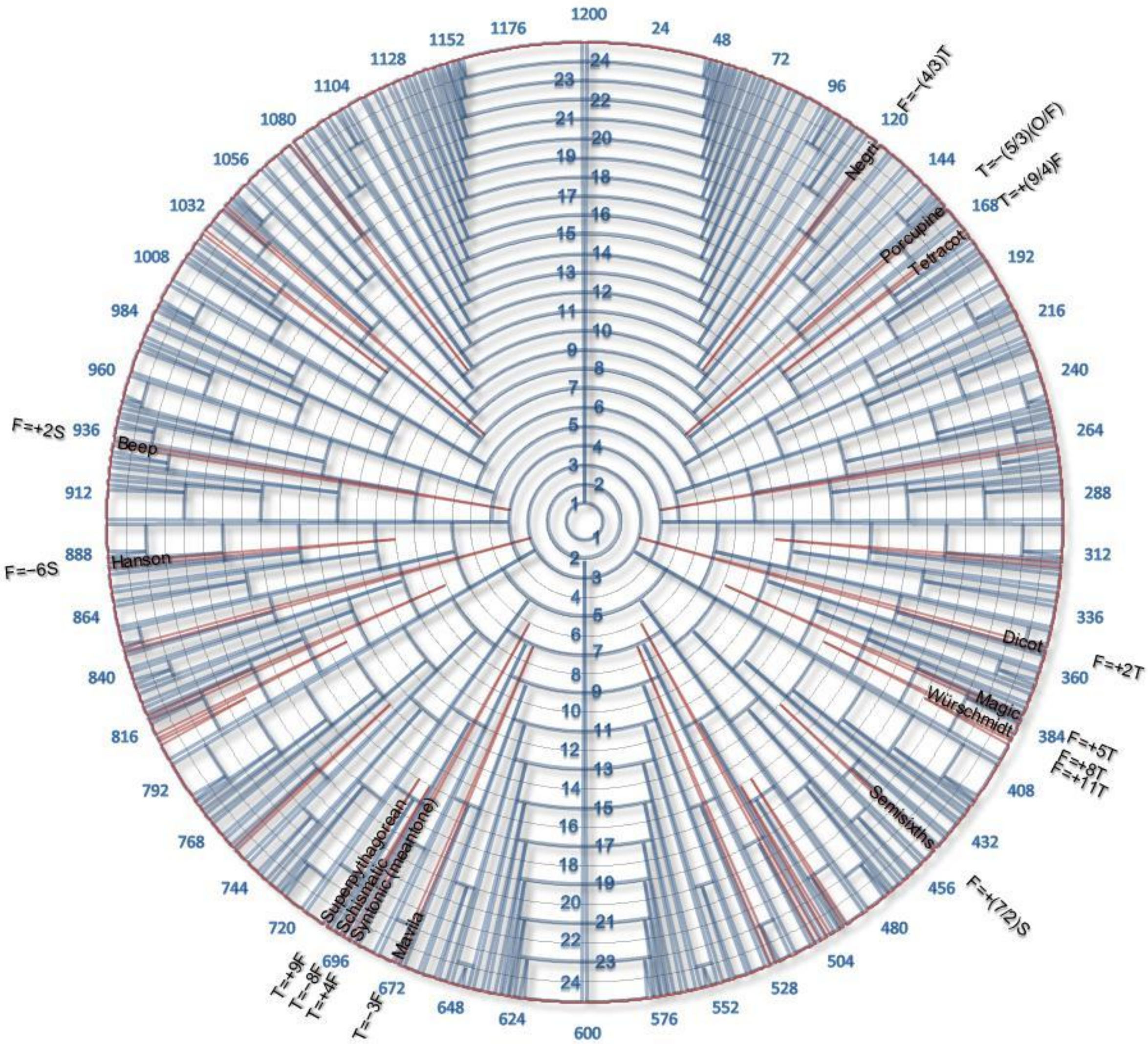
DYNAMIC TONALITY—LEARN MORE (VERSION 14/09/2008)

Please note that this is an unfinished paper, currently in the process of being written. For that reason it may contain mistakes, undefined terms, and dead-end references.



Contents

DYNAMIC TONALITY—INTRODUCTION

Dynamic Tonality is a simple method of controlling the tuning and timbre of musical tones to enable:

1. *tuning continua*—a choice of different mappings from just intonation to scale note, giving a choice of familiar and radically unfamiliar scale forms;
2. *dynamic tuning*—free movement between a number of different tunings including equal temperaments, non-equal temperaments, circulating temperaments, and closely related just intonations;
3. *dynamic timbre*—free movement between *timbres* that are perfectly harmonic and perfectly optimized for the tuning.
4. *invariant fingering*—the fingering pattern for all voice-leading intervals is consistent across all possible keys and tunings (when they are played on any musical controller that has a two-dimensional lattice of buttons or keys)(1);

The section Dynamic Tonality—In Theory describes, and hopefully explains, some of the key concepts behind Dynamic Tonality. The section Dynamic Tonality—The Parameters describes each of the Dynamic Tonality parameters is in detail.

DYNAMIC TONALITY—IN THEORY

Tuning Continua

A *tuning continuum* is a range of tunings over which all possible melodies built from harmonic consonances, and the voice-leading intervals between them, have the same contour (2).

Two-dimensional Tunings

Musically useful continua can be generated by adding and subtracting two intervals of variable size—such as a perfect fifth and an octave, which generate the familiar pentatonic, diatonic, and chromatic scales (3; 4; 5).

Continua generated from quite different pairs of intervals have radically different scale structures (6); *The Viking* implements two non-standard continua: *Magic*, which is generated by a variable "major third" and octave, and *Hanson*, which is generated by a variable "minor third" and octave. Both of these continua produce scales that are relatively unfamiliar, but which still feature numerous well-tuned major and minor triads.

There is an important class of scales called *MOS* (3; 7) or *well-formed* (8) that consist of just two scale step sizes; for example, the pentatonic scale has two large steps and three small, while the diatonic has five large steps (whole-tones) and two small steps (semitones). Many other musically useful scales are similar to MOS in that they consist mostly of the same step intervals (e.g., the harmonic minor scale has three whole-tones, three semitones, and one augmented tone).

MOS scales generated by alternative continua have different numbers of small and large steps. The ratio between the two generators $z = \beta/\alpha$ determines the ratio between the sizes of the large and small steps. For example, when $z = 11/19 = 57.9\%$, the diatonic semitone is two thirds (66.7%) the size of the diatonic whole-tone; when $z = 18/31 = 58.1\%$, the semitone is three fifths (60%) the size of a whole-tone; when $z = 7/12 = 58.3\%$, the semitone is half (50%) the size of the whole-tone; when $z = 10/17 = 58.8\%$, the diatonic semitone is one third (33.3%) the size of the whole-tone. Note how, in the diatonic scale, the ratio z is inversely monotonic to the ratio between the two scale step sizes.

Dynamic Tonality provides two parameters to adjust the current tuning—a z-*slider* (which controls the *relative* sizes of any given scale's steps), and an *octave-width* knob (which controls the *absolute* size of any given scale's steps). Using parameters for α and z (rather than α and β) is particularly important to provide easier manipulation of tunings when the timbre is fully adjusted to match the tuning—see the section Dynamic Timbre for further details.

Related Just Intonations

The above range of tunings can be expanded to include related just intonations. *Just intonation* (*JI*) tuning systems contain many intervals tuned to small number ratios (e.g., 3:2, 4:3, 5:4, 6:5, 7:5, 7:6, etc.), and these intervals are typically thought to be maximally consonant and "in tune" when using sounds with harmonic spectra (i.e., the vast majority of conventional Western instruments). For this reason, just intonation has been frequently cited as an ideal tuning (e.g., by Helmholtz (9), Partch (10), and Mathieu (11)). However, 5-limit just intonation is three-dimensional, and higher-limit JI's have even more dimensions, making it all but impossible to avoid "wolf" intervals when mapping to a fixed pitch instrument (1).

The vertical axis of the Tone Diamond enables the tuning to be moved from a regular two-dimensional temperament towards two possible related just intonations. The reason for two choices is because there is always ambiguity about precisely which JI interval is represented by a tempered interval. For this reason we provide two aesthetically motivated choices: *Major JI*, at the bottom of the diamond, maximizes the number of justly tuned major triads (of ratio 4:5:6); while *Minor JI*, at the top of the diamond, maximizes the number of justly tuned minor triads (of ratio 10:12:15).

Dynamic Tuning

In a musical scale generated by a perfect fifth and an octave (such as the diatonic scale), varying the size of the fifth produces a wide range of useful tunings. For example, string and wind instrument players often prefer the narrow minor seconds of Pythagorean tuning (i.e., perfect fifths tuned to 702 cents) for expressive melodies, and the just major thirds of quarter-comma meantone (i.e., perfect fifths tuned to 696 cents) for euphonious sustained chords (12). There are also numerous "non-standard" tunings used in non-Western music such as the 5-TET of Indonesian Slendro (13), and the 7-TET of traditional Thai (14) and Mandinka balafon music (15).

This means that a musician can use a simple slider or joystick to vary the size of the perfect fifth to mimic the intonation devices used by string and wind players, and easily move between (even radically) different tunings throughout a performance. These benefits of dynamic tuning also apply to all non-standard tuning continua too.

Dynamic Timbre

As described above, scales and tunings can vary significantly throughout a tuning continuum, and even more significantly between different tuning continua. With fingering invariance, these scales and tunings can even be easy to play and learn. However, we have not yet addressed the aesthetic significance of Dynamic Tonality.

There are many reasons why 12-TET is the standard, so why bother with non-standard tunings?

There is strong evidence that an instrument's timbre determines the tunings it plays in best, and that by taking the reverse approach each tuning also has *related* timbres that sound most consonant (16). Most non-standard tunings have no access to a related timbre, due to the limitations of acoustic instruments. Dynamic Tonality addresses this issue by adjusting the timbre to match the current tuning specified by user. More precisely, it allows the user to specify to what degree the *partials* (also known as *overtones* or *harmonics*) should match the tuning.

To achieve this, *The Viking* uses additive synthesis, which generates each partial with its own sinusoidal oscillator. Frequencies are determined automatically by Dynamic Tonality parameters, while amplitudes are determined automatically by a traditional waveform selector and further modified by familiar subtractive synthesis filters and envelopes. *The Viking*, therefore, enables sounds similar to those produced by the "classic" subtractive synthesizers (e.g., Moog, ARP) to be adjusted spectrally.

Any synthesis method that allows for the frequency of each partial to be adjusted in real-time is applicable to Dynamic Tonality. For example, the *TransFormSynth* (built by Bill Sethares) uses spectral analysis and resynthesis to adjust the partials of audio samples and even live audio.

The degree to which the spectrum is adjusted is controllable via the δ (delta) parameter, which is the horizontal axis of the Tone Diamond.

Fingering Invariance

Perhaps one of the most important aspects of Dynamic Tonality is that it allows for fingering invariance. On any musical controller with keys or buttons, intervals are played by a specific set of buttons that outline a geometric shape (see Figure 1 for an example—the *Thummer*'s two-dimensional button lattice). However, on one-dimensional interfaces like the piano keyboard, a given interval often requires different shapes. For example, D–F# and F–A are each major thirds, but form different shapes on a piano keyboard. Contrarily, on a *Thummer*, all major thirds are exactly the same shape; in fact, all intervals on the *Thummer* require only one fingering. Similarly, piano keyboards require multiple fingerings across a tuning continuum, while *Thummers* only require one fingering. This property of invariance across tunings and keys is called *fingering invariance* (1).

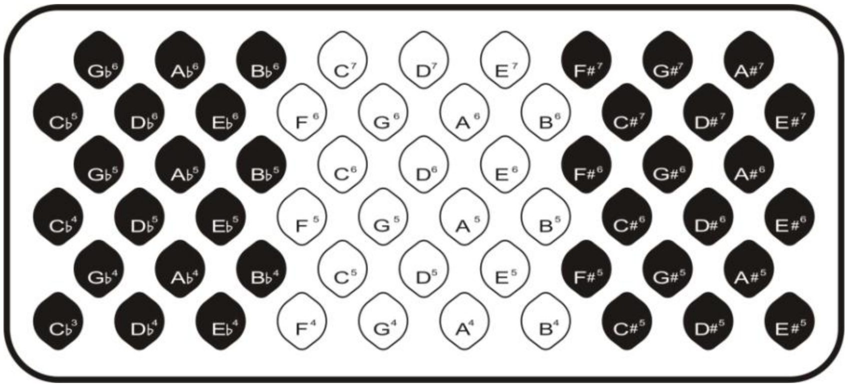


Figure 1. The *Thummer*'s two-dimensional button lattice.

With fingering invariance, musicians need only learn the fingering of a given interval or chord once, and thereafter apply that shape to all occurrences of that interval or chord, independent of its location within a key, across keys, or across tunings. This reduces rote memorization considerably and engages the student's visual and tactile senses in discerning the consistency of musical patterns.

New Tonal Possibilities

The Viking and *TransFormSynth* currently implement two non-standard continua—"Magic" and "Hanson"—which open up interesting compositional avenues. They contain scales that embed numerous major and minor triads, but have a radically different structure to those found in any standard Western tuning. For example, the Magic continuum has a ten-note MOS/well-formed scale (with seven small steps and three large steps) that contains ten major or minor triads; the Hanson continuum has an eleven-note MOS/well-formed scale (with seven small steps and four large steps) that also contains ten major or minor triads. *Magic Traveller* uses the above-described Magic scale. It may well be that the chords in these systems have functional relationships that are quite different to those found in standard diatonic/chromatic tonality. Such systems, therefore, open up the possibility of an aesthetic research program similar to that which may be said to have characterized the development of common-practice from the birth of harmonic tonality in the sixteenth century to the "crisis of tonality" at the end of the nineteenth.

But the well-structured tonal relationships found in these continua do not support only a strictly tonal compositional style. Serial (and other "atonal") compositional techniques are just as applicable to these alternative continua, as are techniques which explore the implications of unusual timbral combinations and structures. Each continuum offers a unique set of mathematical possibilities and constraints. For example, the familiar 12-note division of the octave has many factors (2, 3, 4, and 6), thus enabling interval classes of these sizes to cycle back to the starting note, and modes of limited transposition to be formed. Conversely, a 13-note division of the octave, which can be made to sound quite "in-tune" when the spectrum is tempered to the Magic continuum, has no factors and so contains no modes of limited transposition and no interval cycles. The 15-note division found in Hanson has factors of 3 and 5, suggesting a quite different set of structural possibilities. *ChatterBar* and *Lighthouse* are both non-serial "atonal" pieces—in 53-TET Syntonic and 11-TET Hanson, respectively. Further musical examples can be found in the Multimedia section of this site, and at Bill Sethares' Spectral Tools Homepage.

Alongside these structural possibilities are the dynamic variations in tuning and timbre that can be easily controlled (and even notated) with the α, β, γ, and δ parameters. These enable keyboard players, for the first time, to easily emulate the fluent changes in tuning frequently used by string and wind players for expressive effect, as well as the more radical dynamic alterations in tuning that can be achieved by large movements of the parameters . For example, smooth changes of tuning and timbre are at the core of Bill Sethares' *C2ShiningC*, while in *Shred*, the music switches from 12-TET to 5-TET Syntonic.

It is to be hoped, therefore, that Dynamic Tonality opens up a rich seam of compositional possibilities that is both deep and accessible.

Dynamic Tonality—The Parameters

z-slider

The z-slider parameter controls the ratio β/α, and therefore controls the relative size of the large and small steps of any MOS scale, such as the Syntonic continuum's pentatonic and diatonic scales (or similar non-MOS scales, like the harmonic minor or ascending melodic minor).

At any z-tuning that is a rational number, in reduced form m/n (for $m, n \in \mathbb{N}$), the tuning is an n-EDα (a tuning with n equal divisions per α). For example, when $z = 7/12$, the tuning is 12-EDα; when $z = 4/7$, the tuning is 7-EDα; when $z = 10/17$, the tuning is 17-EDα; etc..

When the spectrum of a sound is fully tempered to the tuning (i.e., the Tone Diamond's control dot is on the *Max. Consonance* line), z-tunings that produce n-TETs with a low-valued n, produce tones that are inherently consonant. For example, in the Syntonic continuum with the Tone Diamond set to *Max. Consonance*, individual tones produced at 7-TET, 12-TET, 17-TET, and 5-TET are inherently consonant (i.e., they are at local dissonance minima, as shown in Figure 2).

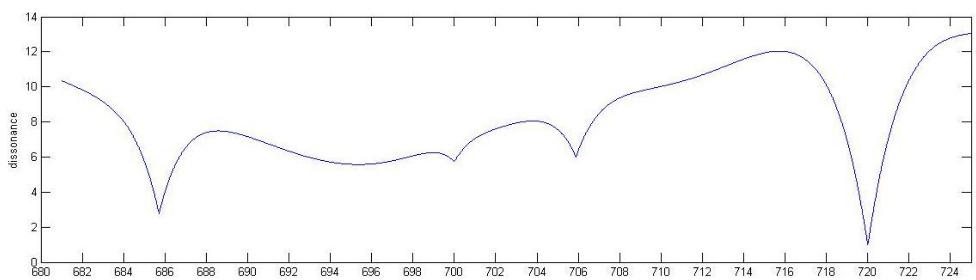


Figure 2. Inherent tonal dissonance for a sawtooth spectrum over the Syntonic tuning continuum ($\alpha = 1200¢$, $686¢ < \beta < 720¢$).

For any given z-tuning, the ratio of MOS and MOS-like scales' step sizes are invariant, and the inherent dissonance for any fully tempered tone is also invariant. It is to preserve these invariances that Dynamic Tonality parameterizes z, rather than b. For example, a musician may have selected fully tempered spectrum and set the z-tuning to 5-TET. the octave-width control can now be changed, thereby stretching the whole scale, but not affecting the inherent consonance, or the ratio of large and small steps (i.e., the tuning will remain a 5-tone equal division of whatever octave width is actually chosen).

The values shown on the z-slider are *relative cents*, as opposed to the more familiar *absolute cents*. A relative cent is 1200th of the octave, even if that octave is tempered. For example, if the octave has a size of 1210 absolute cents and the z-tuning is 720 relative cents, then the absolute cents value of beta is $(720/1200) * 1210 = 726$.

Octave-width

The *octave-width* parameter controls the size of the α. If α is a direct mapping of the just octave 2/1, then the octave-width directly controls the size of α. In some tuning continua, however, the octave may be produced by a multiple of α. For example, in some continua, the octave is produced by 2α (where α is tuned to approximately 600 absolute cents); in some continua the octave is produced by 3α (where α is tuned to approximately 400 absolute cents). The octave-width control, therefore, controls the absolute cents size of the value $r\alpha$.

The reason that Dynamic Tonality parameterizes octave-width ($r\alpha$) rather than α, is because the octave (i.e., any interval tuned to approximately 1200 absolute cents) is an almost universally used interval (17) representing a sort of pitch equivalence (18; 19) with high affinity (20) and a high level of consonance, whereas equal divisions of the octave are not necessarily perceptually meaningful tuning "landmarks".

In conjunction with a constant z-tuning, adjusting octave-width can be used to "stretch" any given scale whilst keeping the relative sizes of its step intervals invariant, and also ensures that inherent tonal dissonance remains invariant across octave "stretchings" and "squashings".

Tone Diamond

2-D Tunings and Related JIs

The vertical dimension of the Tone Diamond moves the tuning smoothly between two-dimensional tuning systems and two possible related just intonations (for more details, see the section Tuning Continua)—the former are found on the central horizontal line marked *Max. Regularity*, the latter are found at the top and bottom diamond points marked *Major JI* and *Minor JI*.

The reason that there are two possible just intonations is that there is always ambiguity about precisely which JI interval is represented by a tempered interval so we provide two aesthetically motivated choices: *Major JI*, at the bottom of the diamond, maximizes the number of justly tuned major triads (of ratio 4:5:6); while *Minor JI*, at the top of the diamond, maximizes the number of justly tuned minor triads (of ratio 10:12:15). Generally speaking there is only one note whose tuning differs between these two possibilities.

The Tone Diamond and z-tuning slider facilitate dynamic tuning changes between many different tuning systems. When the Tone Diamond's control point is anywhere along the central horizontal line (the *Max. Regularity* line), the tuning is a regular one- or two-dimensional tuning such as 12-TET or quarter-comma meantone, as shown on the main tuning slider. When the control point is moved upwards or downwards the tuning moves towards a related just intonation. The tunings that are intermediate between perfect regularity and JI are like the circulating temperaments of Kirnberger and Vallotti in that every key has a (slightly) different tuning. And all of these tunings have essentially the same fingering when played on a 2-D lattice controller.

Spectral Tempering

The Tone Diamond also facilitates the dynamic tempering of spectrum. When the timbre of a sound is fully tempered to match the underlying tuning, the consonance of tempered consonances is maximized, whatever the tuning. This means that tunings that sound highly dissonant with conventional harmonic spectra (such as 5-TET) can sound perfectly consonant once the spectrum has been adjusted to match (check the latter half of *Shred* for an example of 5-TET with fully tempered spectra). Spectral tempering is not transparent—perfectly harmonic tones have a uniquely "pure" sounding pitch (and timbral quality), while some tempered spectra can sound inherently dissonant and "dirty", while others may sound "complex", "bell-like", or "interesting". Also, as shown in Figure 2, when the spectrum is fully tempered, tunings that produce low n-TETs with low value n's are, themselves, inherently consonant, and therefore pleasing to listen too but also complex (a good example of this being the 5-TET spectrum in the second half of *Shred*). For full details on the precise mathematical methods used to achieve the matching of timbre and tuning over various tuning continua please see (21).

The horizontal dimension of the Tone Diamond controls how much of this tempering is applied—when the Tone Diamond's control dot is anywhere on the *Max. Harmonicity* line, the sound remains harmonic with integer partials; whenever the control dot is on the *Max. Consonance* line, the partials are always tempered so that they are fully related to the tuning.

The Tone Diamond is labeled to show that the further the control point is from the *Max. Harmonicity* line, the less harmonic its partials; the further the control point is from the *Max. Consonance* line, the less related its partials are to the tuning; the further the control point is from the *Max. Regularity* line, the less regular are its interval sizes. The diamond clearly illustrates how every possible position of the control point represents a compromise between maximal harmonicity, maximal consonance, and maximal regularity; no system can have all three at the same time.

Tuning Continuum

There are numerous possible tuning continua. At present *The Viking* and *TransFormSynth* implement just three—Syntonic, Magic, and Hanson. We expect to add many more.

The choice of tuning continuum effects the range of *z*-tunings available on the slider, and the way that the partials of the sounds are tempered to match the tuning. For any given continuum, there are further options that affect the way that partials are tempered—these are the Prime Limit controls.

Prime-limit Controls

A tone with harmonic partials contains partials at integer multiples of the fundamental—if the fundamental has a frequency of 100Hz, there will be partials at 200, 300, 400, 500, 600, 700, 800, 900, 1000, 1100, 1200, 1300, ...Hz. A tone with 11-limit partials contains only those harmonics that are multiples of the fundamental that are divisible by primes up to 11 (i.e., the primes 2, 3, 5, 7, and 11).

When *11-limit* is chosen, the synthesizer tones are 11-limit, and all of these partials can be tempered to relate to the current tuning.

When *7-limit fixed* is chosen, the tones are 11-limit, but only the 7-limit partials are tempered to match the tuning—the 11-limit partials are always untempered and are perfectly harmonic.

When *7-limit off* is chosen, the 11-limit partials are switched off, leaving just the 7-limit partials, all of which can be tempered to relate to the current tuning.

When *5-limit fixed* is chosen, the tones are 11-limit, but only the 5-limit partials are tempered to match the tuning—the 7- and 11-limit partials are always untempered and are perfectly harmonic.

When *5-limit off* is chosen, the 7- and 11-limit partials are switched off, leaving just the 7-limit partials, all of which can be tempered to relate to the current tuning.

NON-DT IMPLEMENTATIONS

Further Resources

Where It All Started

Milne, Andrew J., Sethares, William, A. and Plamondon, James. *The X_System.* 2006.

Peer-reviewed Publications About Dynamic Tonality

Isomorphic controllers and dynamic tuning: Invariant fingering over a tuning continuum. **Milne, Andrew J., Sethares, William A. and Plamondon, James.** 4, Winter 2007, Computer Music Journal, Vol. 31, pp. 15–32.

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Spectral tools for Dynamic Tonality and audio morphing. **Sethares, William A., et al.** Forthcoming, Computer Music Journal, Under review.

Relating Timbre and Tuning

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Regular Temperaments, MOS Scales, Tuning Theory

A middle path between just intonation and the equal temperaments. **Erlich, Paul.** 18, 2006, Xenharmonicon, pp. 159–199.

Alternate Tunings Mailing List.

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